

Analytical study on holographic superconductors with backreactions

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Abstract

We employ the variational method for the Sturm-Liouville eigenvalue problem to analytically investigate the properties of the holographic superconductors. We find that the analytic method is still powerful when the backreaction is turned on. Reducing step size in the iterative procedure, we observe that the consistency of results between the analytic and numerical computations can be further improved. The obtained analytic result can be used to back up the numerical computations in the holographic superconductor in the fully backreacted spacetime.

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I. INTRODUCTION

As a powerful tool to analyse strongly coupled quantum field theories, the anti-de Sitter/conformal field theories (AdS/CFT) correspondence [1–3] states that a d -dimensional weakly coupled dual gravitational description in the bulk is equivalent to a $(d - 1)$ -dimensional strongly coupled conformal field theory on the boundary. In recent years, this principle has been used to provide some meaningful theoretical insights in order to understand the physics of high T_c superconductors from the gravitational dual [4, 5]. It was found that the spontaneous $U(1)$ symmetry breaking by bulk black holes can be used to construct gravitational duals of the transition from normal state to superconducting state in the boundary theory, which exhibits the behavior of the superconductor [6]. Due to the potential applications to the condensed matter physics, there have been a lot of works studying various gravity models with the property of the so-called holographic superconductor (for reviews, see Refs. [7–9] and references therein).

In most cases, the studies on the holographic superconductors focus on the probe approximation where the backreaction of matter fields on the spacetime metric is neglected. When taking the backreaction into account, it was found that even the uncharged scalar field can form a condensate in the $(2 + 1)$ -dimensional holographic superconductor model [5]. Furthermore, in the p-wave holographic dual models, it was argued that the phase transition that leads to the formation of vector hair changes from the second order to the first order when the gravitational coupling is large enough [10, 11]. There have been accumulated interest to study the holographic superconductor away from the probe limit [12–23]. Recently, the effect of the backreaction has been investigated between holographic insulator and superconductor [24–28].

Almost all works on the holographic dual models away from the probe limit were based on numerical computations. In order to back up numerical results and gain more insights in the effect of the backreaction, a fully analytic study is called for. Refining the analytic matching method developed in [29, 30], Kanno calculated the critical temperature of $(3 + 1)$ -dimensional holographic superconductors in Einstein-Gauss-Bonnet gravity with backreaction and found that the backreaction makes condensation harder [31]. This analytic approach has been extended to derive the critical magnetic field in holographic superconductors with backreaction [32]. However, the analytic matching method can keep valid only when the matching point is chosen within an appropriate range in higher dimensions ($d > 6$) [30]. Moreover, when the scalar mass is zero in the Gauss-Bonnet holographic superconductors, the curvature correction term does not contribute to

the analytic approximation, which leads the analytic procedure to break down [30]. Recently, Siopsis and Therrien developed a new analytic method. They extended the variational method for the Sturm-Liouville (S-L) eigenvalue problem to analytically calculate the critical exponent near the critical temperature and found that the analytical results obtained by this way are in good agreement with the numerical findings [33]. Considering the effectiveness and accuracy of the S-L method, many authors have used it to analytically investigate the properties of holographic superconductors in AdS black hole backgrounds [34–41] and soliton backgrounds [42–45]. But these attempts were limited in the probe limit. It was argued that the S-L method is more effective for the analytic study of the condensation than the matching method [42]. It is of interest to examine whether the S-L method is still valid to explore the holographic superconductivity when the backreaction is turned on. This is not trivial since the analytic study can help to confirm the numerical result. Furthermore it can clearly disclose the influence of the role of the backreaction in the condensation. In this work, we will generalize the variational method for the S-L eigenvalue problem to study holographic superconductor away from the probe limit.

The organization of the work is as follows. In Sec. II, we will introduce the holographic superconductor models with backreactions in the d -dimensional AdS black hole background. In Sec. III we will give an analytical investigation of the holographic superconductors by using the S-L method. We will conclude in the last section of our main results.

II. HOLOGRAPHIC SUPERCONDUCTOR MODELS WITH BACKREACTIONS

We begin with the general action describing a charged, complex scalar field in the d -dimensional Einstein-Maxwell action with negative cosmological constant

$$S = \int d^d x \sqrt{-g} \left[\frac{1}{2\kappa^2} (R - 2\Lambda) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - |\nabla\psi - iqA\psi|^2 - m^2|\psi|^2 \right], \quad (1)$$

where $\kappa^2 = 8\pi G_d$ is the d -dimensional gravitational constant, $\Lambda = -(d-1)(d-2)/(2L^2)$ is the cosmological constant, A and ψ represent the gauge field and a scalar field with charge q respectively. Since we are interested in including the backreaction, we will take the metric ansatz for the d -dimensional planar black hole

$$ds^2 = -f(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{f(r)} + r^2 h_{ij} dx^i dx^j, \quad (2)$$

where f and χ are functions of r only, $h_{ij}dx^i dx^j$ denotes the line element of a $(d-2)$ -dimensional hypersurface with the curvature $k = 0$. The Hawking temperature of this black hole, which will be interpreted as the

temperature of the CFT, is given by

$$T_H = \frac{f'(r_+)e^{-\chi(r_+)/2}}{4\pi}, \quad (3)$$

where the prime denotes a derivative with respect to r . r_+ is the black hole horizon determined by $f(r_+) = 0$.

We consider the electromagnetic field and the scalar field in the forms

$$A = \phi(r)dt, \quad \psi = \psi(r), \quad (4)$$

where without loss of generality $\psi(r)$ can be taken to be real. Thus, from the variation of the action with respect to the matter and metric we obtain the equations of motion

$$\chi' + \frac{4\kappa^2 r}{d-2} \left(\psi'^2 + \frac{q^2 e^\chi \phi^2 \psi^2}{f^2} \right) = 0, \quad (5)$$

$$f' - \left[\frac{(d-1)r}{L^2} - \frac{(d-3)f}{r} \right] + \frac{2\kappa^2 r}{d-2} \left[m^2 \psi^2 + \frac{1}{2} e^\chi \phi'^2 + f \left(\psi'^2 + \frac{q^2 e^\chi \phi^2 \psi^2}{f^2} \right) \right] = 0, \quad (6)$$

$$\phi'' + \left(\frac{d-2}{r} + \frac{\chi'}{2} \right) \phi' - \frac{2q^2 \psi^2}{f} \phi = 0, \quad (7)$$

$$\psi'' + \left(\frac{d-2}{r} - \frac{\chi'}{2} + \frac{f'}{f} \right) \psi' - \frac{m^2}{f} \psi + \frac{q^2 e^\chi \phi^2}{f^2} \psi = 0. \quad (8)$$

It should be noted that the transformation $\tilde{\phi} = \phi/q$ and $\tilde{\psi} = \psi/q$ in the action (1) does not change the form of the Maxwell and the scalar equations, but the gravitational coupling in the Einstein equation changes $\kappa^2 \rightarrow \kappa^2/q^2$. Thus, the probe limit is equivalent to letting $q \rightarrow \infty$. Without loss of generality, we can set $q = 1$ and keep κ^2 finite when we take the backreaction into account [16, 18, 21, 31].

For the normal phase, $\psi(r) = 0$, we find that χ is a constant and the analytic solutions to Eqs. (6) and (7) lead to the AdS Reissner-Nordström black holes with the metric coefficient

$$f = \frac{r^2}{L^2} - \frac{1}{r^{d-3}} \left[\frac{r_+^{d-1}}{L^2} + \frac{(d-3)\kappa^2 \rho^2}{(d-2)r_+^{d-3}} \right] + \frac{(d-3)\kappa^2 \rho^2}{(d-2)r^{2d-6}}, \quad \phi = \mu - \frac{\rho}{r^{d-3}}, \quad (9)$$

where μ and ρ are interpreted as the chemical potential and charge density in the dual field theory respectively.

When $\kappa = 0$, the metric coefficient f goes back to the case of the Schwarzschild AdS black hole.

In order to get the solutions in superconducting phase, where $\psi(r) \neq 0$, we have to count on the appropriate boundary conditions. At the horizon r_+ , the metric functions χ and f satisfy

$$\begin{aligned} \chi'(r_+) &= -\frac{4\kappa^2 r_+}{d-2} \left[\psi'(r_+)^2 + \frac{e^{\chi(r_+)} \phi'(r_+)^2 \psi(r_+)^2}{f'(r_+)^2} \right], \\ f'(r_+) &= \frac{(d-1)r_+}{L^2} - \frac{2\kappa^2 r_+}{d-2} \left[m^2 \psi(r_+)^2 + \frac{1}{2} e^{\chi(r_+)} \phi'(r_+)^2 \right], \end{aligned} \quad (10)$$

and the regularity condition gives the boundary conditions

$$\phi(r_+) = 0, \quad \psi(r_+) = \frac{f'(r_+)\psi'(r_+)}{m^2}. \quad (11)$$

At the asymptotic AdS boundary ($r \rightarrow \infty$), the asymptotic behaviors of the solutions are

$$\chi \rightarrow 0, \quad f \sim \frac{r^2}{L^2}, \quad \phi \sim \mu - \frac{\rho}{r^{d-3}}, \quad \psi \sim \frac{\psi_-}{r^{\Delta_-}} + \frac{\psi_+}{r^{\Delta_+}}, \quad (12)$$

where the exponent Δ_{\pm} is defined by $[(d-1) \pm \sqrt{(d-1)^2 + 4m^2}]/2$. Notice that, provided Δ_- is larger than the unitarity bound, both ψ_- and ψ_+ can be normalizable and they can be used to define operators on the dual field theory, $\psi_- = \langle \mathcal{O}_- \rangle$, $\psi_+ = \langle \mathcal{O}_+ \rangle$, respectively [4, 5]. For simplicity, we will scale $L = 1$ in the following calculation.

III. ANALYTICAL INVESTIGATION OF THE HOLOGRAPHIC SUPERCONDUCTORS

Here we will apply the S-L method [33] to analytically investigate the properties of the s-wave holographic superconductor phase transition with backreactions. We will derive the relation between the critical temperature T_c and charge density ρ near the phase transition point and examine the effect of the backreaction.

Introducing a new variable $z = r_+/r$, we can rewrite the Einstein, Maxwell and the scalar equations into

$$\chi' - \frac{4\kappa^2}{d-2} \left(z\psi'^2 + \frac{r_+^2}{z^3 f^2} e^{\chi} \phi^2 \psi^2 \right) = 0, \quad (13)$$

$$f' - \frac{(d-3)f}{z} + \frac{(d-1)r_+^2}{L^2 z^3} - \frac{2\kappa^2 r_+^2}{(d-2)z^3} \left[m^2 \psi^2 + \frac{z^4}{2r_+^2} e^{\chi} \phi'^2 + f \left(\frac{z^4}{r_+^2} \psi'^2 + \frac{1}{f^2} e^{\chi} \phi^2 \psi^2 \right) \right] = 0, \quad (14)$$

$$\phi'' + \left(\frac{\chi'}{2} - \frac{d-4}{z} \right) \phi' - \frac{2r_+^2 \psi^2}{z^4 f} \phi = 0, \quad (15)$$

$$\psi'' - \left(\frac{\chi'}{2} + \frac{d-4}{z} - \frac{f'}{f} \right) \psi' - \frac{r_+^2}{z^4} \left(\frac{m^2}{f} - \frac{e^{\chi} \phi^2}{f^2} \right) \psi = 0, \quad (16)$$

where the prime now denotes the derivative with respect to z .

Since the value of the scalar operator $\langle \mathcal{O}_+ \rangle$ (or $\langle \mathcal{O}_- \rangle$) is small near the critical point, we can introduce it as an expansion parameter

$$\epsilon \equiv \langle \mathcal{O}_i \rangle, \quad (17)$$

with $i = +$ or $i = -$. Note that we are interested in solutions where ψ is small, therefore from Eqs. (15) and (16) we can expand the scalar field ψ and the gauge field ϕ as [31, 32, 46]

$$\begin{aligned} \psi &= \epsilon \psi_1 + \epsilon^3 \psi_3 + \epsilon^5 \psi_5 + \dots, \\ \phi &= \phi_0 + \epsilon^2 \phi_2 + \epsilon^4 \phi_4 + \dots, \end{aligned} \quad (18)$$

where $\epsilon \ll 1$. The metric function $f(z)$ and $\chi(z)$ can be expanded around the Reissner-Nordström AdS spacetime

$$\begin{aligned} f &= f_0 + \epsilon^2 f_2 + \epsilon^4 f_4 + \dots, \\ \chi &= \epsilon^2 \chi_2 + \epsilon^4 \chi_4 + \dots. \end{aligned} \quad (19)$$

For the chemical potential μ , we will allow it to be corrected order by order [46]

$$\mu = \mu_0 + \epsilon^2 \delta\mu_2 + \dots, \quad (20)$$

where $\delta\mu_2 > 0$. Thus, near the phase transition, we find a result for the order parameter as a function of the chemical potential

$$\epsilon \approx \left(\frac{\mu - \mu_0}{\delta\mu_2} \right)^{1/2}, \quad (21)$$

whose critical exponent $\beta = 1/2$ is the universal result from the Ginzburg-Landau mean field theory of phase transitions. Obviously, the order parameter becomes zero and phase transition can happen if $\mu \rightarrow \mu_0$, which shows that the critical value of μ is $\mu_c = \mu_0$.

At the zeroth order, we can get the solution ϕ_0 from Eq. (15), i.e., the electromagnetic field behaves like $\phi_0(z) = \mu_0(1 - z^{d-3})$, which gives a relation $\mu_0 = \rho/r_+^{d-3}$. At the critical point μ_c , we can find $\mu_0 = \mu_c = \rho/r_{+c}^{d-3}$, where r_{+c} is the radius of the horizon at the critical point. In order to use the analytical S-L method [33], we will set

$$\phi_0(z) = \lambda r_{+c}(1 - z^{d-3}), \quad (22)$$

with $\lambda = \rho/r_{+c}^{d-2}$. Inserting this solution into Eq. (14), we obtain the metric function

$$f_0(z) = r_+^2 g(z) = r_+^2 \left[\frac{1}{L^2 z^2} - \frac{z^{d-3}}{L^2} - \frac{(d-3)\kappa^2 \lambda^2}{d-2} z^{d-3} (1 - z^{d-3}) \right], \quad (23)$$

where we define a new function $g(z)$ for simplicity in the following calculation.

At the first order, the asymptotic AdS boundary conditions ($z \rightarrow 0$) for ψ can be expressed as

$$\psi_1 \sim \frac{\psi_-}{r_+^{\Delta_-}} z^{\Delta_-} + \frac{\psi_+}{r_+^{\Delta_+}} z^{\Delta_+}. \quad (24)$$

So we introduce a trial function $F(z)$ near the boundary $z = 0$ [33]

$$\psi_1(z) \sim \frac{\langle O_i \rangle}{r_+^{\Delta_i}} z^{\Delta_i} F(z), \quad (25)$$

where we have imposed the boundary condition $F(0) = 1$ and $F'(0) = 0$. Substituting Eq. (25) into Eq. (16), we obtain the equation of motion for $F(z)$

$$F'' + \left[\frac{2(\Delta_i + 2) - d}{z} + \frac{g'}{g} \right] F' + \left[\frac{\Delta_i}{z} \left(\frac{\Delta_i + 3 - d}{z} + \frac{g'}{g} \right) + \frac{\lambda^2(1 - z^{d-3})^2}{z^4 g^2} - \frac{m^2}{z^4 g} \right] F = 0. \quad (26)$$

In order to simplify the following calculation, we will express the backreacting parameter κ as

$$\kappa_n = n\Delta\kappa, \quad n = 0, 1, 2, \dots, \quad (27)$$

where $\Delta\kappa = \kappa_{n+1} - \kappa_n$ is the step size of our iterative procedure. Considering the fact that $\kappa^2 \lambda^2 = \kappa_n^2 \lambda^2 = \kappa_n^2 (\lambda^2|_{\kappa_{n-1}}) + 0[(\Delta\kappa)^4]$ (note that we have set $\kappa_{-1} = 0$ and $\lambda^2|_{\kappa_{-1}} = 0$), we will use the following form of $g(z)$ in our discussion

$$g(z) \approx \frac{1}{L^2 z^2} - \frac{z^{d-3}}{L^2} - \frac{(d-3)\kappa_n^2 (\lambda^2|_{\kappa_{n-1}})}{d-2} z^{d-3} (1 - z^{d-3}), \quad (28)$$

where $\lambda^2|_{\kappa_{n-1}}$ is the value of λ^2 for κ_{n-1} . After defining a function which obeys

$$T(z) = z^{2\Delta_i+1} [(d-2)(z^{1-d} - 1) - (d-3)L^2 \kappa_n^2 (\lambda^2|_{\kappa_{n-1}})(1 - z^{d-3})], \quad (29)$$

we can convert Eq. (26) to be

$$(TF')' + T \left[\frac{\Delta_i}{z} \left(\frac{\Delta_i + 3 - d}{z} + \frac{g'}{g} \right) + \frac{\lambda^2(1 - z^{d-3})^2}{z^4 g^2} - \frac{m^2}{z^4 g} \right] F = 0. \quad (30)$$

From the Sturm-Liouville eigenvalue problem [47], we write down the expression which can be used to estimate the minimum eigenvalue of λ^2

$$\lambda^2 = \frac{\int_0^1 T (F'^2 - U F^2) dz}{\int_0^1 T V F^2 dz}, \quad (31)$$

with

$$U = \frac{\Delta_i}{z} \left(\frac{\Delta_i + 3 - d}{z} + \frac{g'}{g} \right) - \frac{m^2}{z^4 g},$$

$$V = \frac{(1 - z^{d-3})^2}{z^4 g^2}. \quad (32)$$

In order to use the variation method, we will assume the trial function to be $F(z) = 1 - az^2$, where a is a constant.

Using Eq. (31) to compute the minimum eigenvalue of λ^2 for $i = +$ or $i = -$, we can obtain the critical temperature T_c for different strength of the backreaction κ and the mass of the scalar field m from the following relation

$$T_c = \frac{1}{4\pi} \left[(d-1) - \frac{(d-3)^2}{d-2} \kappa_n^2 (\lambda^2|_{\kappa_{n-1}}) \right] \left(\frac{\rho}{\lambda} \right)^{\frac{1}{d-2}}. \quad (33)$$

As an example, we calculate the case for $d = 5$ and $m^2 L^2 = -3$ with the chosen values of the backreaction parameter κ for $i = +$, i.e., $\Delta_+ = 3$. Setting $\Delta\kappa = 0.05$, for $\kappa_0 = 0$ we have

$$\lambda^2 = \frac{2(-18 + 27a - 14a^2)}{6(4\ln 2 - 3) + 16(3\ln 2 - 2)a + (24\ln 2 - 17)a^2}, \quad (34)$$

whose minimum is $\lambda^2|_{\kappa_0} = 18.23$ at $a = 0.7218$. According to Eq. (33), we can easily get the critical temperature $T_c = 0.1962\rho^{1/3}$, which agrees well with the numerical result $T_c = 0.1980\rho^{1/3}$ [48]. For $\kappa_1 = 0.05$, substituting $\lambda^2|_{\kappa_0}$ into Eqs. (28) and (29) we obtain

$$\lambda^2 = \frac{4.466 - 6.682a + 3.462a^2}{0.1714 - 0.1600a + 0.04592a^2}, \quad (35)$$

which attains its minimum $\lambda^2|_{\kappa_1} = 18.11$ at $a = 0.7195$. Hence the critical temperature reads $T_c = 0.1934\rho^{1/3}$, which is also in good agreement with the numerical result $T_c = 0.1953\rho^{1/3}$. For $\kappa_2 = 0.10$, putting $\lambda^2|_{\kappa_1}$ in Eqs. (28) and (29) we arrive at

$$\lambda^2 = \frac{4.364 - 6.478a + 3.349a^2}{0.1740 - 0.1633a + 0.04705a^2}, \quad (36)$$

whose minimum is $\lambda^2|_{\kappa_2} = 17.75$ at $a = 0.7122$. So the critical temperature is $T_c = 0.1852\rho^{1/3}$, which is again consistent with the numerical finding $T_c = 0.1874\rho^{1/3}$. For other values of κ , the similar iterative procedure also can be applied to present the analytic result for the critical temperature. When we reduce the step size, for example to fix $\Delta\kappa = 0.025$, we can also compute the critical temperature T_c in the similar way. In Table I we give the critical temperature T_c for the scalar operator $\langle \mathcal{O}_+ \rangle$ when we fix the mass of the scalar field $m^2 L^2 = -3$ for different strength of the backreaction by choosing the step size $\Delta\kappa = 0.05$ and 0.025 , respectively. We find that the analytic results derived from the S-L method are in very good agreement with the numerical calculation. Furthermore we observe that when we reduce the step size $\Delta\kappa$, we can improve the analytic result and get the critical temperature more consistent with the numerical result.

TABLE I: The critical temperature T_c with the chosen values of the backreaction parameter κ and the step size $\Delta\kappa$ for the condensates of the scalar operator $\langle \mathcal{O}_+ \rangle$ in the case of 5-dimensional AdS black hole background. Here we fix the mass of the scalar field by $m^2 L^2 = -3$.

	Analytical($\Delta\kappa = 0.05$)	Analytical($\Delta\kappa = 0.025$)	Numerical
$\kappa = 0$	$0.1962\rho^{1/3}$	$0.1962\rho^{1/3}$	$0.1980\rho^{1/3}$
$\kappa = 0.05$	$0.1934\rho^{1/3}$	$0.1934\rho^{1/3}$	$0.1953\rho^{1/3}$
$\kappa = 0.10$	$0.1852\rho^{1/3}$	$0.1853\rho^{1/3}$	$0.1874\rho^{1/3}$
$\kappa = 0.15$	$0.1718\rho^{1/3}$	$0.1722\rho^{1/3}$	$0.1748\rho^{1/3}$
$\kappa = 0.20$	$0.1540\rho^{1/3}$	$0.1549\rho^{1/3}$	$0.1580\rho^{1/3}$
$\kappa = 0.25$	$0.1330\rho^{1/3}$	$0.1345\rho^{1/3}$	$0.1382\rho^{1/3}$
$\kappa = 0.30$	$0.1098\rho^{1/3}$	$0.1123\rho^{1/3}$	$0.1165\rho^{1/3}$

For completeness, we also extend the investigation to the 4-dimensional AdS black hole background. In Table II, we present the critical temperature T_c of the chosen parameter κ with the scalar operators $\langle \mathcal{O}_- \rangle$

and $\langle \mathcal{O}_+ \rangle$ for the $(2+1)$ -dimensional superconductor if we fix the mass of the scalar field by $m^2 L^2 = -2$ and the step size by $\Delta\kappa = 0.05$. The agreement of the analytic results derived from S-L method with the numerical calculation shown in Tables I and II is impressive.

TABLE II: The critical temperature T_c obtained by the analytical S-L method (left column) and from numerical calculation (right column) with the chosen values of the backreaction parameter κ for the condensates of the scalar operators $\langle \mathcal{O}_- \rangle$ and $\langle \mathcal{O}_+ \rangle$ in the case of 4-dimensional AdS black hole background. Here we fix the mass of the scalar field by $m^2 L^2 = -2$ and the step size by $\Delta\kappa = 0.05$.

	$\langle \mathcal{O}_- \rangle$		$\langle \mathcal{O}_+ \rangle$	
$\kappa = 0$	$0.2250\rho^{1/2}$	$0.2255\rho^{1/2}$	$0.1170\rho^{1/2}$	$0.1184\rho^{1/2}$
$\kappa = 0.05$	$0.2249\rho^{1/2}$	$0.2253\rho^{1/2}$	$0.1163\rho^{1/2}$	$0.1177\rho^{1/2}$
$\kappa = 0.10$	$0.2246\rho^{1/2}$	$0.2250\rho^{1/2}$	$0.1141\rho^{1/2}$	$0.1156\rho^{1/2}$
$\kappa = 0.15$	$0.2241\rho^{1/2}$	$0.2245\rho^{1/2}$	$0.1106\rho^{1/2}$	$0.1121\rho^{1/2}$
$\kappa = 0.20$	$0.2235\rho^{1/2}$	$0.2239\rho^{1/2}$	$0.1057\rho^{1/2}$	$0.1074\rho^{1/2}$
$\kappa = 0.25$	$0.2226\rho^{1/2}$	$0.2230\rho^{1/2}$	$0.0998\rho^{1/2}$	$0.1017\rho^{1/2}$
$\kappa = 0.30$	$0.2216\rho^{1/2}$	$0.2220\rho^{1/2}$	$0.0929\rho^{1/2}$	$0.0951\rho^{1/2}$

It further supports the observation obtained first in the numerical computation that the stronger backreaction can make the scalar hair more difficult to be developed [12–23]. The consistency between the analytic and numerical results indicates that the S-L method is a powerful analytic way to investigate the holographic superconductor even when we take the backreaction into account.

IV. CONCLUSIONS

We have generalized the variational method for the Sturm-Liouville eigenvalue problem to analytically investigate the properties of the holographic superconductor with backreactions. We found that in the fully backreacted spacetime, the S-L method is still powerful to disclose the property of the condensation. Our analytic results are in very good agreement with those obtained from numerical computations. If we reduce the step in the iterative procedure, we can further improve our analytic results and improve the consistency with the numerical findings. Our analytic result shows that the backreaction makes the critical temperature of the superconductor decrease, which can be used to back up the numerical finding as shown in figure 2 of Ref. [5] that the backreaction can hinder the condensation to be formed.

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